ECET 380-Numerical Methods-Project 2 – Week 2

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Introduction

This week's project concerns itself with two pipes in parallel. Last project you learned that pumps provide a head gain. Conversely, pipes provide a head loss, and act very much like a resistor. The goal of this project is to determine the parameters of two pipes in parallel, with one pipe's diameter varying.

Piping characteristics

The purpose of a pipe is to contain and guide a flowing fluid. A pipe is characterized by the length L, diameter D, and roughness. Usually the pipe is of circular cross section.

Fluid Characteristics and Properties

Mass Flow Rate

The mass flow rate is the amount of mass passing through a cross sectional area. The units of mass flow rate are mass/time, and is calculated as

$$\dot{m} = \rho A v$$

Where

ρ=density (kg/m^3)

A=cross sectional area of pipe (m^2)

v=average speed of fluid in pipe (m/s)

Reynolds Number

The Reynolds number, denoted by Re, is the ratio of the fluid inertial forces to the viscous forces. It is a unitless number, and is often used in fluid mechanics to determine the transport status of the fluid (laminar or turbulent). The Reynolds number is calculated by

$$Re = \frac{\rho v D}{\mu}$$

Where

ρ=density (kg/m^3)

v=average speed of fluid in pipe (m/s)

D=Hydraulic diameter of pipe (for circular pipes, it's just the pipe diameter) (m/s)

μ=Fluid dynamic viscosity (Pa-s)

Head loss in a pipe

When a fluid travels through a pipe, there is a pressure drop due to fluid friction. The conversion of flow energy to internal energy due to fluid friction is called *head loss*. The units of head loss is length, and is calculated using the Darcy equation:

$$h_L = f\frac{L}{D}\frac{v^2}{2g}$$

Where

L=pipe length (m)

v=average speed of fluid in pipe (m/s)

D=Hydraulic diameter of pipe (for circular pipes, it's just the pipe diameter) (m/s)

g=gravitational acceleration constant (m/s^2)

f =Darcy friction factor (unitless)

When pipes are in series, the heal loss is additive. Moreover, for parallel pipes, the head loss is the same for each pipe. This is equivalent to voltage drops in resistors.

The friction factor

How does one calculate the friction factor? The answer to this question is nontrivial. It took a lot of research and experimentation to determine the contributions to the friction factor. Eventually the friction factor was put into a graph in the form of a *Moody diagram*, shown in Figure 1.

How does one calculate the friction factor using the Moody diagram? To first order, there are two variables needed in the calculation: the relative roughness ε/D and the Reynolds number. First, locate the value of the relative roughness on the right vertical axis. Follow the solid black line representing the relative roughness (interpolate if you have to) until you get to the Reynolds number of interest. From there travel directly to the left until you intersect the left vertical axis. The corresponding value at that location is the friction factor.

As an example, consider the case where the relative roughness is 0.002 and the Reynolds number is 20,000. From 0.002, follow the solid black line until you get to the Reynolds number of 20,000. Traveling directly to the left yields a friction factor of 0.03.

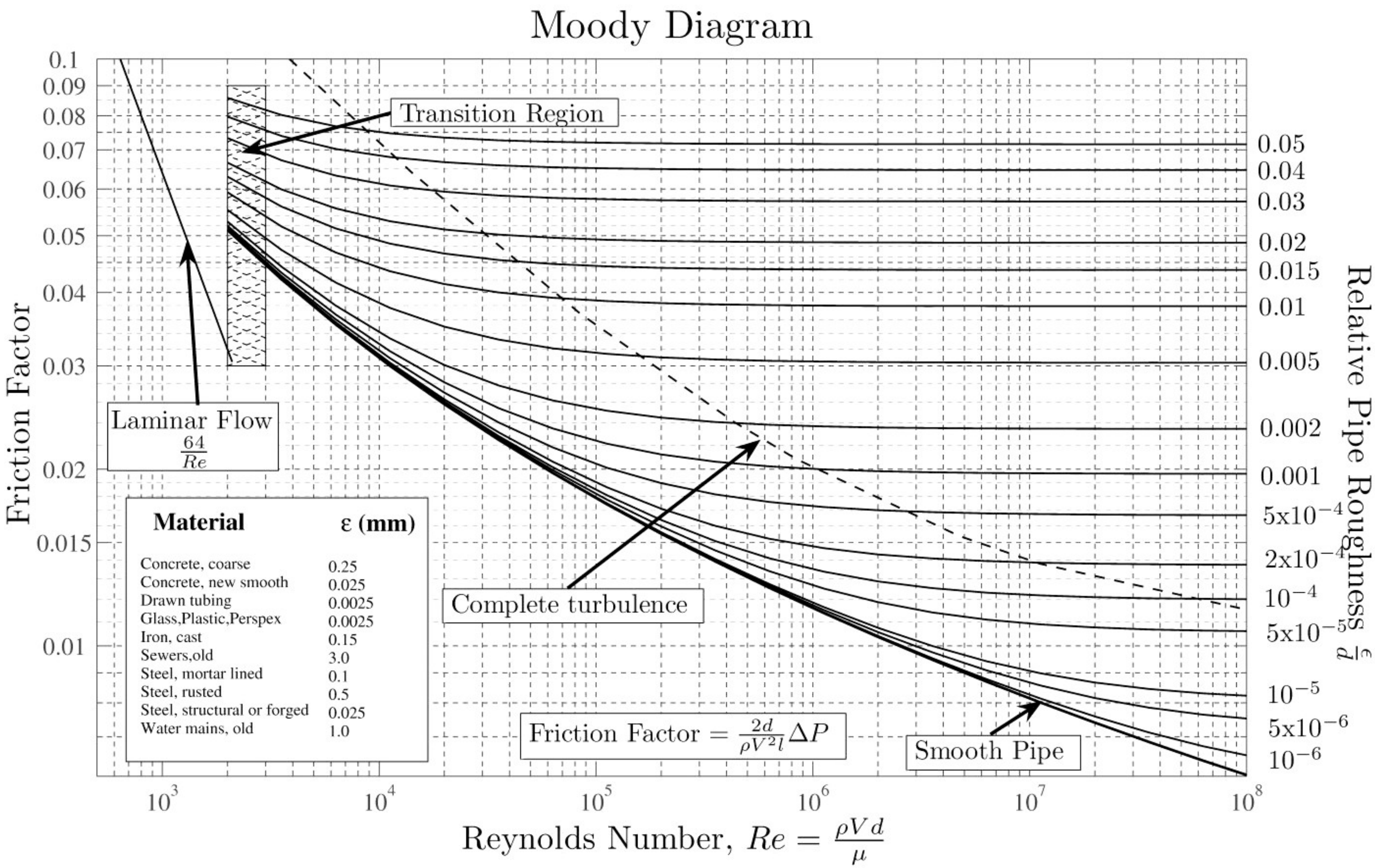

Figure 1 Moody diagram

Analytic approximation for the Moody diagram

The Moody diagram shows three regions of interest in terms of fluid transport characteristics:

1. Laminar region ($Re < 2300$) – the fluid travels very smoothly throughout the pipe. In this region, the friction factor is calculated by $f = 64/Re$
2. Critical/Transition region ($2300 \leq Re < 4000$) – the fluid is not fully developed into completely turbulent flow. Characterization of the friction factor in this region is not trivial. However, we will assume there is a linear relationship between the laminar region (with $Re = 2300$) and the turbulent region ($4000 \leq Re$). Use the relationship corresponding to each region and linearly interpolate.
3. Turbulent region ($Re \geq 4000$) – the fluid mixes as it travels through the pipe. The friction factor in this region can be approximated via the Colebrook equation:
$$\frac{1}{\sqrt{f}} = -2log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

Solving the Colebrook equation numerically

The Colebrook equation is a transcendental equation: the independent variable f cannot be isolated and directly solved. However, we don't need an exact solution of the friction factor: a value that is close enough to the truth (within an acceptable tolerance) can be solved numerically. If we rearrange the Colebrook equation, we get

$$\frac{1}{\sqrt{f}} + 2log_{10}\left(\frac{\frac{\epsilon}{D}}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) = 0$$

This can be solved. If we define the equation

$$y = \frac{1}{\sqrt{f}} + 2log_{10}\left(\frac{\frac{\epsilon}{D}}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

The friction factor can be found by solving for the root of y. One easy algorithm to find the friction factor is to sweep the value of the friction factor from a lower value to an upper value. The value of f which yields the lowest absolute value is the best approximation for the friction factor. However, this is NOT the method you will use in the project.

While there are a multitude of algorithms to solve for the root of y, we focus on the Newton-Raphson method. This method uses the current values of $f, y(f)$ and $y'(f)$:

$$f_{new} = f_{current} - \frac{y(f_{current})}{y'(f_{current})}$$

This process is iterative, and terminates when the approximate error is below the stopping threshold, or when the maximum number of iterations has been reached. It is up to the student to determine the derivative y'.

The Project Itself

This project is concerned with two pipes in parallel. In this configuration, the head loss from both pipes is the same:

$$h_{L,pipe\ 1} = h_{L,pipe\ 2}$$

For this project, these two pipes will be identical in every aspect EXCEPT the diameter of pipe 2. You will be varying the diameter of pipe 2, and observing the fluid parameters of each pipe at each of these diameters. The parameters of the pipes are:

Parameter	Symbol	Units	Pipe 1	Pipe 2
Length	L	m	10	10
Diameter	D	m	0.2	varying
Roughness	ε	mm	0.25	0.25
Fluid density	ρ	kg/m^3	1000	1000
Dynamic Viscosity	μ	Pa-s	0.001	0.001

The diameter of pipe 2 will vary from 0.03 m to 0.5 m in 0.01 m increments. The total mass flow rate entering the system is 5 kg/s. The goal of this project is to determine the mass flow rate through each pipe, as well as other characteristics for each configuration. While this may seem trivial at the surface, the nonlinearity behavior of the friction factor can affect the results significantly.

What you need to do

You need to provide the following graphs, with the independent axis being the diameter of pipe 2, and the results of both pipes on the same graph:

1. Mass flow rate
2. Friction factor
3. Reynolds number
4. Head loss

Additionally, you need to provide another graph that plots the friction factor versus Reynolds number for each pipe. This should be done on a semilog axis (if using MATLAB, use the *semilogx* command). If done properly, something very interesting can be observed.

For the friction factor calculation, the stopping criteria should be an approximate relative error of 0.001%, and a maximum iteration value of 1000. Moreover, you should use the Newton-Raphson method to solve the friction factor (the derivative of the function needs to be calculated by the student).

To determine the proper mass flow rate through each pipe, you will be taking a very brute force approach. For each pipe configuration, the mass flow rate in pipe 1 starts at 0.01 kg/s. The head loss calculations for each pipe are then calculated. The difference between the head loss results is the imbalance in the system. Thus, you need to keep incrementing the mass flow rate through pipe one until the absolute value of the imbalance is minimized. So the absolute value of the error will decrease until it meets the minimum. If you do this properly, it will only take one iteration pass the minimum to determine when to terminate the search. The mass flow rate before the increased absolute imbalance is the minimum.

Tips for solving this project

1. Use anonymous (inline) functions in MATLAB, or lambda functions in Python whenever possible
2. You are needing to find the mass flow rate through each pipe. This means you need to find the solution to $\left|h_{L,pipe\ 1} - h_{L,pipe\ 2}\right|$ is minimized
3. Create a separate function that calculates the friction factor, with the inputs being the Reynolds number and relative roughness. You can validate your code works by comparing the results to the Moody diagram provided
4. Use object oriented programming by creating a pipe class. This can make your project significantly easier. If you do this, data can be accesses simply. Suggested input values stored in an object should are
    a. Pipe length
    b. Pipe diameter
    c. Pipe roughness
    d. Pipe mass flow
    e. Fluid density
    f. Fluid viscosity

    Suggested calculated values stored in the object are:
    g. Pipe cross sectional area
    h. Fluid velocity in pipe
    i. Reynolds number of the fluid in the pipe
    j. Relative roughness of the pipe
    k. Pipe friction factor
    l. Pipe head loss

Remember to convert the diameter to mm when calculating the relative roughness

What should be in your report

The following sections should be in your report:

1. Introduction
2. Theory of pipe behavior (note: Do NOT go into detail in this section, but rather a basic introduction to the theory, such as noted above)
3. Theory of Newton-Raphson method
4. System configuration and parameters
5. Graphical results
6. Discussion
7. Appendix that displays your code you wrote